Int J Theor Phys (2007) 46: 3247–3253
DOI 10.1007/s10773-007-9443-x

Propagation of Photons in Spacetime

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Received: 28 March 2007 / Accepted: 1 June 2007 / Published online: 15 June 2007

Abstract A quantum field theory QED formalism is systematically developed to describe photon propagation in *spacetime* as a time evolution process based on the actual *physical* process of propagation between emitters and detectors as applied to the reflection of photons. This development, as well as early studies by Feynman, clearly show that a practical, computational and predictive dynamical formalism in *spacetime* was lacking. The present one *generalizes* to different experimental situations and *other* interacting field theories as well emphasizing the practicality of the problem treated here.

Keywords Photon dynamics in spacetime and time evolution · QED and field theories in spacetime

Much progress has been done over the years [1, 2] to describe, especially quantum theoretically, the localization of photons in space [3]. It is fair to say, however, that there is still no explicit dynamical, non-heuristic, actual quantum (field) theory QED formalism worked out, as dictated by the latter, to describe the propagation of photons in *spacetime* in explaining even a simplest experiment as the reflection of photons off a reflecting surface as a time evolution process. This is certainly remarkable in the progress of physics, knowing that QED has been around for sometime and, as Feynman ([4], p. 3) puts it, it has been thoroughly analyzed, in his legendary Alix G. Mautner Memorial Lectures. The latter fascinating, though heuristic treatment [4] in words is, of course, far from a definite theoretical description but, in spite being addressed to non-specialists, the discussion clearly indicates, and as the present analysis shows, that a theoretical formalism, as stated above, to explain a simplest experiment in *spacetime* in a quantum (field) theory QED setting is lacking. For one thing, the amplitude of propagation of photons in *spacetime*, as a time evolution process, in infinitely extended space, for example, from a point x_1^μ to a point x_2^μ turns out to be given by $(\mathrm{i}/(\pi)^2)(x_2^0 - x_1^0)/[(x_2 - x_1)^2]^2$ rather than by the familiar Feynman propagator

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$\mathrm{i}/(x_2 - x_1)^2$, with the former satisfying a key completeness relation for the internal consistency of the theory as formulated in spacetime. The purpose of this work is to develop such a formalism in detail based on the actual *physical process of the propagation of photons from emitters to detectors* as obtained from the so-called vacuum-to-vacuum transition amplitude [5–12] for the underlying theory. This method has been quite successful over the years in the easiness of momentum space computations of physical processes, avoiding of introducing so-called wavefunctions, not to mention of the elegance of the formalism as opposed to more standard techniques, and at the same time gaining much physical insight as particles propagate from emitters, interact, and finally particles reach the detectors as occurring in practice. The present analysis rests on three general key points: (i) By working directly in spacetime for the vacuum-to-vacuum transition amplitude, for given boundary conditions (B.C.), and from the expressions of the amplitudes for the emission and detection of photon excitations by the external sources, an amplitude of propagation between different spacetime points from emitters to detectors, causally arranged, is extracted and, as mentioned above, it does *not* coincide with the Feynman propagator for the corresponding B.C. This step already shows the power of determining amplitudes of propagation by introducing external sources. (ii) The amplitude of propagation is shown to satisfy a *completeness* relation as photons propagate between different points critical for the internal consistency of the theory in spacetime. (iii) Application of these amplitudes to describe in detail the experiment being sought by showing, in the process, very rapid exponential damping beyond the classical point of impact for the corresponding amplitude of occurrence. The reader will soon realize that our theoretical quantum (field) theory QED formalism is reduced to a non-operator approach and opens a way to describe, as a time evolution process, photon dynamics in *spacetime* and *other* field theory interactions in different experimental situations as well.

Let $|0_\mp\rangle$ denote the vacuum states before/after the external current $J^\mu(x)$, coupled to the vector potential $A_\mu(x)$ in Maxwell's Lagrangian, is switched on/off. The boundary conditions taken are $\langle 0_+|\mathbf{E}_\parallel(x)|0_-\rangle = \mathbf{0}$, $\langle 0_+|\mathbf{B}_\perp(x)|0_-\rangle = \mathbf{0}$ for $z \to +0$, where the reflecting surface is taken to consist the $x^1 - x^2$ plane, with $x^3 \equiv z \geq 0$, and $\mathbf{E}_\parallel/\mathbf{B}_\perp$ denote the components of the electric/magnetic fields parallel/perpendicular to the $x^1 - x^2$ plane. The vacuum-to-vacuum transition amplitude $\langle 0_+|0_-\rangle^J$ is then given by [11]

$$\langle 0_+|0_-\rangle^J = \mathrm{e}^{\frac{\mathrm{i}}{2}\int (\mathrm{d}x_1)(\mathrm{d}x_2) J^\mu(x_2) D'_{\mu\nu}(x_2,x_1) J^\nu(x_1)} \tag{1}$$

where invoking the conservation law $\partial_\mu J^\mu = 0$, the photon propagator in half-space may be written as

$$D'_{\mu\nu}(x_2, x_1) = \int \frac{(\mathrm{d}Q)}{(2\pi)^4} \frac{\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}_2-\mathbf{r}_1)}\mathrm{e}^{-\mathrm{i}Q^0(x_2^0-x_1^0)}}{Q^2 - \mathrm{i}\varepsilon} \times [g_{\mu\nu}\mathrm{e}^{\mathrm{i}q(z_2-z_1)} + (-g_{\mu\nu} + 2g_{\mu 3}g_{\nu 3})\mathrm{e}^{-\mathrm{i}q(z_2+z_1)}] \tag{2}$$

$\varepsilon \to +0$, $x = (x^0, \mathbf{r}, z)$, $Q = (Q^0, \mathbf{k}, q)$ with $\mathbf{r}$ lying in the $x^1 - x^2$ plane. Since $J^\mu(x)$, by definition, vanishes for $z \leq 0$, we may integrate over all spacetime points in (1). Gauge invariance of the theory as well as the positivity condition $|\langle 0_+|0_-\rangle^J|^2 \leq 1$ are readily established [11]. We consider a causal arrangement, $J^\mu(x) = J_1^\mu(x) + J_2^\mu(x)$, of two currents with $J_2^\mu(x)$, the detector, switched on after $J_1^\mu(x)$, the emitter, is switched off. By invoking the condition $\partial_\mu J^\mu = 0$, we may then write

$$\langle 0_+|0_-\rangle^J = \langle 0_+|0_-\rangle^{J_2}\, \mathrm{e}^{\mathrm{i}\Omega}\, \langle 0_+|0_-\rangle^{J_1} \tag{3}$$

where, with $i, j = 1, 2, 3$, $x = (x^0, \mathbf{r}, z)$, $x' = (x^0, \mathbf{r}, -z)$,

$$\Omega = \int (\mathrm{d}x_1)(\mathrm{d}x_2)\mathrm{i}J^i_{2\mathrm{T}}(x_2)[-\mathrm{i}\Delta_+(x_2, x_1)\delta^{ij} \\ - \mathrm{i}\Delta_+(x'_2, x_1)(-\delta^{ij} + 2\delta^{i3}\delta^{j3})]\mathrm{i}J^j_{1\mathrm{T}}(x_1) \tag{4}$$

and for $x^0_2 > x^0_1$,

$$-\mathrm{i}\Delta_+(x_2, x_1) = \int \frac{\mathrm{d}^3\mathbf{Q}}{(2\pi)^3 2Q^0}\,\mathrm{e}^{\mathrm{i}Q(x_2-x_1)}, \qquad Q^0 = |\mathbf{Q}|, \tag{5}$$

$$J^{\mathrm{i}}_{\mathrm{T}}(x) = \int \frac{(\mathrm{d}Q)}{(2\pi)^4}\,\mathrm{e}^{\mathrm{i}Qx} J^i_{\mathrm{T}}(Q) \tag{6}$$

and for $Q^0 = |\mathbf{Q}|$, $Q^i J^i_{\mathrm{T}}(Q) = 0$. The second term within the square brackets in (4) corresponds to a non-trivial transition.

Now we use the identity

$$-\mathrm{i}\Delta_+(x_4, x_1) = \int' \mathrm{d}^3\mathbf{x}_2 \int' \mathrm{d}^3\mathbf{x}_3 \int' \mathrm{d}^3\mathbf{x} D_>(x_4, x_3) \\ \times [D(x_3, x)\mathrm{i}\overleftrightarrow{\partial_0} D(x, x_2)]D_<(x_2, x_1) \tag{7}$$

in (4), where $\overleftrightarrow{\partial_0} = \overrightarrow{\partial_0} - \overleftarrow{\partial_0}$, $x^0_4 > x^0_3 > x^0_2 > x^0_1$, $\int' \mathrm{d}^3\mathbf{x} = \int_{\mathbb{R}^2} \mathrm{d}^2\mathbf{r} \int_0^\infty \mathrm{d}z$, and

$$D_>(x_4, x_3) = \int \frac{\mathrm{d}^3\mathbf{Q}}{4\pi^3\sqrt{2Q^0}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}_4-\mathbf{r}_3)}\mathrm{e}^{-\mathrm{i}Q^0(x^0_4-x^0_3)}\mathrm{e}^{\mathrm{i}qz_4}\sin qz_3, \tag{8}$$

$$D_<(x_2, x_1) = \int \frac{\mathrm{d}^3\mathbf{Q}}{4\pi^3\sqrt{2Q^0}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}_2-\mathbf{r}_1)}\mathrm{e}^{-\mathrm{i}Q^0(x^0_2-x^0_1)}\sin qz_2 \mathrm{e}^{-\mathrm{i}qz_1}, \tag{9}$$

$$D(x_2, x_1) = \int \frac{\mathrm{d}^3\mathbf{Q}}{2\pi^3\sqrt{2Q^0}} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot(\mathbf{r}_2-\mathbf{r}_1)}\mathrm{e}^{-\mathrm{i}Q^0(x^0_2-x^0_1)}\sin qz_2 \sin qz_1. \tag{10}$$

Given two real unit 3-vectors $\mathbf{n} = (a, b, c) \equiv \mathbf{n}_+$, $\mathbf{n}' = (a, b, -c) \equiv \mathbf{n}_-$, we introduce two sets of unit 3-vectors $(\mathbf{e}_1, \mathbf{e}_2)$, $(\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2)$ by $\mathbf{e}_1 = \mathbf{n}\times\mathbf{n}'/|\mathbf{n}\times\mathbf{n}'| = \boldsymbol{\epsilon}_1$, $\mathbf{e}_2 = \mathbf{n}\times\mathbf{e}_1$, $\boldsymbol{\epsilon}_2 = \mathbf{n}'\times\boldsymbol{\epsilon}_1$, satisfying $\mathbf{n}_+\cdot\mathbf{e}_\lambda = 0$, $\mathbf{n}_-\cdot\boldsymbol{\epsilon}_\lambda = 0$ for $\lambda = 1, 2$. We use the completeness relations

$$\delta^{ij} = n^i_+ n^j_+ + \Sigma_\lambda e^i_\lambda e^j_\lambda = n^i_- n^j_- + \Sigma_\lambda \epsilon^i_\lambda \epsilon^j_\lambda \tag{11}$$

and also set

$$\mathbf{S}_\pm(x) = \int \frac{\mathrm{d}^3\mathbf{Q}}{4\pi^3\sqrt{2Q^0}} \mathbf{S}_\pm(Q_\pm)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}}\mathrm{e}^{-\mathrm{i}Q^0x^0}\sin(\pm qz) \tag{12}$$

with $Q_+ = Q$, $Q_- = Q' = (Q^0, \mathbf{k}, -q)$, $Q^0 = |\mathbf{Q}|$,

$$\mathbf{S}_\pm(Q_\pm) = \mathbf{J}_{\mathrm{T}}(Q_\pm) - \frac{\mathbf{Q}_\pm + Q^0\mathbf{n}_\pm}{Q^0 + \mathbf{n}\cdot\mathbf{Q}}\,\mathbf{n}_\pm\cdot\mathbf{J}_{\mathrm{T}}(Q_\pm) \tag{13}$$

from which we have

$$S_{\pm}^{*i}(Q_{\pm})\delta^{ij}S_{\pm}^{j}(Q_{\pm}) = J_{\mathrm{T}}^{*i}(Q_{\pm})\delta^{ij}J_{\mathrm{T}}^{j}(Q_{\pm}), \tag{14}$$

$$S_{\mp}^{*i}(Q_{\mp})[-\delta^{ij} + 2\delta^{i3}\delta^{j3}]S_{\pm}^{j}(Q_{\pm}) = J_{\mathrm{T}}^{*i}(Q_{\mp})[-\delta^{ij} + 2\delta^{i3}\delta^{j3}]J_{\mathrm{T}}^{j}(Q_{\pm}). \tag{15}$$

Here we note that $\mathbf{n}_{\pm}\cdot\mathbf{Q}_{\pm} = \mathbf{n}\cdot\mathbf{Q}$, and that for the points $\mathbf{Q}_{\pm} = -\mathbf{n}_{\pm}|\mathbf{Q}|$, not only the numerators in the second term in (13) vanish but also $\mathbf{n}_{\pm}\cdot\mathbf{J}_{\mathrm{T}}(Q_{\pm}) = 0$. Hence these points are apparent singularities in (12) belonging to sets of measure zero. Note, in particular, that $\mathbf{n}_{\pm}\cdot\mathbf{S}_{\pm}(x) = 0$.

From (4), (11–15), the following explicit expression for Ω emerges

$$\begin{aligned}\Omega = \int' \mathrm{d}^3\mathbf{x}_1 \int' \mathrm{d}^3 x_2 \frac{\nabla_{+}(x_2, x_1)}{2} \sum_{\lambda} [&(\mathrm{i}\mathbf{S}_{+}^{*}(x_2)\cdot\mathbf{e}_{\lambda})(\mathrm{i}\mathbf{S}_{+}(x_1)\cdot\mathbf{e}_{\lambda})\\ &+ (\mathrm{i}\mathbf{S}_{-}^{*}(x_2)\cdot\boldsymbol{\epsilon}_{\lambda})(\mathrm{i}\mathbf{S}_{-}(x_1)\cdot\boldsymbol{\epsilon}_{\lambda}) + (-1)^{\lambda}(\mathrm{i}\mathbf{S}_{-}^{*}(x_2)\cdot\boldsymbol{\epsilon}_{\lambda})(\mathrm{i}\mathbf{S}_{+}(x_1)\cdot\mathbf{e}_{\lambda})\\ &+ (-1)^{\lambda}(\mathrm{i}\mathbf{S}_{+}^{*}(x_2)\cdot\mathbf{e}_{\lambda})(\mathrm{i}\mathbf{S}_{-}(x_1)\cdot\boldsymbol{\epsilon}_{\lambda})]\end{aligned} \tag{16}$$

with $\nabla_{+}(x_2, x_1) = \int' \mathrm{d}^3\mathbf{x} D(x_3, x)\mathrm{i}\overleftrightarrow{\partial_0} D(x, x_1)$. Clearly, the last two terms in (16) correspond to non-trivial transitions.

Let $|\mathbf{e}_{\lambda}, \mathbf{n}_{+}, x\rangle \equiv |\lambda, +, x\rangle$, $|\boldsymbol{\epsilon}_{\lambda}, \mathbf{n}_{-}, x\rangle \equiv |\lambda, -, x\rangle$ denote photon excitation states emitted at spacetime point $x = (x^0, \mathbf{r}, z)$ with associated vectors $(\mathbf{e}_{\lambda}, \mathbf{n}_{+})$, $(\boldsymbol{\epsilon}_{\lambda}, \mathbf{n}_{-})$, respectively. The physical significance of these associated vectors will be discussed in the light of the experiment being sought. A unitarity expansion of $\langle 0_{+}|0_{-}\rangle^{J}$ will include, in particular, the following four terms describing the emission, propagation and detection of photon excitations:

$$\begin{aligned}&\langle 0_{+}|\lambda, +, x_2\rangle^{J_2}\langle\lambda, +, x_2|\alpha, +, x_1\rangle\langle\alpha, +, x_1|0_{-}\rangle^{J_1},\\ &\langle 0_{+}|\lambda, -, x_2\rangle^{J_2}\langle\lambda, -, x_2|\alpha, +, x_1\rangle\langle\alpha, +, x_1|0_{-}\rangle^{J_1},\\ &\langle 0_{+}|\lambda, -, x_2\rangle^{J_2}\langle\lambda, -, x_2|\alpha, -, x_1\rangle\langle\alpha, -, x_1|0_{-}\rangle^{J_1},\\ &\langle 0_{+}|\lambda, +, x_2\rangle^{J_2}\langle\lambda, +, x_2|\alpha, -, x_1\rangle\langle\alpha, -, x_1|0_{-}\rangle^{J_1}.\end{aligned} \tag{17}$$

Here, for example, $\langle\alpha, +, x_1|0_{-}\rangle^{J_1}$ denotes the amplitude for the emission of a photon excitation in state $|\alpha, +, x_1\rangle$, with associated vectors $\mathbf{e}_{\alpha}, \mathbf{n}_{+}$, and $\langle 0_{+}|\lambda, -, x_2\rangle^{J_2}$ denotes the amplitude for the detection of a photon excitation in state $|\lambda, -, x_2\rangle$ with associated vectors $\boldsymbol{\epsilon}_{\lambda}, \mathbf{n}_{-}$. Most importantly $\langle\lambda, -, x_2|\alpha, +, x_1\rangle$, for example, denotes the amplitude of propagation of a photon excitation from spacetime point x_1 and associated vectors $\mathbf{e}_{\alpha}, \mathbf{n}_{+}$, to a spacetime point x_2 and ending up with associated vectors $\boldsymbol{\epsilon}_{\lambda}, \mathbf{n}_{-}$.

Upon comparing the four terms in (17) with the corresponding ones in Ω given in (16), *and* using the completeness relation

$$\sum_{\delta=\pm}\int' \mathrm{d}^3\mathbf{x}\,\langle\lambda, \delta_2, x_2|\lambda, \delta, x\rangle\langle\lambda, \delta, x|\lambda, \delta_1, x_1\rangle = \langle\lambda, \delta_2, x_2|\lambda, \delta_1, x_1\rangle \tag{18}$$

with $\delta_1, \delta_2 = \pm$, we obtain

$$\langle 0_{+}|\lambda, +, x\rangle^{J_2} = (\mathrm{i}\mathbf{S}_{+}^{*}(x)\cdot\mathbf{e}_{\lambda})\langle 0_{+}|0_{-}\rangle^{J_2}, \tag{19}$$

$$\langle 0_{+}|\lambda, -, x\rangle^{J_2} = (\mathrm{i}\mathbf{S}_{-}^{*}(x)\cdot\boldsymbol{\epsilon}_{\lambda})\langle 0_{+}|0_{-}\rangle^{J_2}, \tag{20}$$

$$\langle\lambda, +, x|0_{-}\rangle^{J_1} = (\mathrm{i}\mathbf{S}_{+}(x)\cdot\mathbf{e}_{\lambda})\langle 0_{+}|0_{-}\rangle^{J_1}, \tag{21}$$

$$\langle\lambda, -, x|0_{-}\rangle^{J_1} = (\mathrm{i}\mathbf{S}_{-}(x)\cdot\boldsymbol{\epsilon}_{\lambda})\langle 0_{+}|0_{-}\rangle^{J_1}, \tag{22}$$

$$\langle \lambda, \pm, x_2 | \alpha, \pm, x_1 \rangle = \frac{1}{2} \delta_{\lambda\alpha} \nabla_+(x_2, x_1), \tag{23}$$

$$\langle \lambda, \pm, x_2 | \alpha, \mp, x_1 \rangle = \frac{(-1)^\lambda}{2} \delta_{\lambda\alpha} \nabla_+(x_2, x_1). \tag{24}$$

See also Eqs. (95), (96) in [12] for definitions of absorption and emission amplitudes of photon excitations corresponding to the ones in (19–22). Note the factor 1/2 in (23), (24) which is essential to satisfy the completeness relation (18). $\nabla_+(x_2, x_1)$ works out to be

$$\nabla_+(x_2, x_1) = \frac{\mathrm{i}}{\pi^2} (x_2^0 - x_1^0) \sum_{\kappa=\pm 1} \frac{\kappa}{[(\mathbf{r}_2 - \mathbf{r}_1)^2 + (z_2 - \kappa z_1)^2 - (x_2^0 - x_1^0)^2]^2} \tag{25}$$

not coinciding with the Feynman propagator for the corresponding B.C., which upon using the Schwinger representation

$$\frac{1}{A^2} = -\int_0^\infty s \mathrm{d}s \mathrm{e}^{-\mathrm{i}s(A - \mathrm{i}\varepsilon)}, \quad \varepsilon \to +0 \tag{26}$$

is conveniently expressed as

$$\nabla_+(x_2, x_1) = \frac{(x_2^0 - x_1^0)}{\mathrm{i}\pi^2} \sum_{\kappa=\pm 1} \kappa \int_0^\infty s \mathrm{d}s \mathrm{e}^{-\mathrm{i}s[(\mathbf{r}_2 - \mathbf{r}_1)^2 + (z_2 - \kappa z_1)^2 - (x_2^0 - x_1^0)^2 - \mathrm{i}\varepsilon]}. \tag{27}$$

In the sequel we suppress the $\mathrm{i}\varepsilon$ factor to simplify the notation.

The transition amplitude that a photon excitation in a state $|\lambda_1, \delta_1, x_1\rangle$ propagates from $x_1 = (x_1^0, \mathbf{r}_1, z_1)$, reaches the reflecting surface within a skin depth, specified by a scale parameter σ and described by a Gaussian density distribution $\mathrm{e}^{-z^2/\sigma^2}/2\sqrt{\pi}\sigma, 0 \leq z$, and ends up in a state $|\lambda_2, -\delta_1, x_2\rangle$ at $x_2 = (x_2^0, \mathbf{r}_2, z_2)$ is given from (16), (18) to be $(x_1^0 < x^0 < x_2^0)$

$$\mathcal{A}(x_2, x_1) = \int_{R^2} \mathrm{d}^2\mathbf{r} \int_0^\infty \mathrm{d}z \frac{\mathrm{e}^{-z^2/\sigma^2}}{2\sqrt{\pi}\sigma} \times \sum_{\delta=\pm} \delta_{\lambda_1\lambda_2} \langle \lambda_2, -\delta_1, x_2 | \lambda_1, \delta, x \rangle \langle \lambda_1, \delta, x | \lambda_1, \delta_1, x_1 \rangle \tag{28}$$

suppressing, for the moment, the indices λ_1, δ_1 in $\mathcal{A}(x_2, x_1)$ to which we will return later. We note from (23), (24), (27), that the z-integrand in (28) is even in z. We may also introduce a surface *density* amplitude $f(\mathbf{R})$ by

$$\mathcal{A}(x_2, x_1) = \int \mathrm{d}^2\mathbf{R} \, f(\mathbf{R}) \tag{29}$$

by multiplying the $\mathbf{r}$-integrand in (28) by the identity

$$\int \frac{\mathrm{d}^2\mathbf{R}}{\pi\sigma_o^2} \mathrm{e}^{-(\mathbf{R} - \mathbf{r})^2/\sigma_o^2} = 1 \tag{30}$$

valid for any $\sigma_o^2 > 0$, and any $\mathbf{r}$, giving from (23), (24), (28), (27)

$$f(\mathbf{R}) = \frac{\delta_{\lambda_1\lambda_2}(-1)^{\lambda_1}}{8} \int \frac{\mathrm{d}^2\mathbf{r}}{\pi\sigma_o^2} \mathrm{e}^{-(\mathbf{r} - \mathbf{R})^2/\sigma_o^2}$$

$$\times \int_{-\infty}^{\infty} \frac{\mathrm{d}z}{\sqrt{\pi}\sigma} \mathrm{e}^{-z^2/\sigma^2} \nabla_+(x_2, x)\nabla_+(x, x_1). \tag{31}$$

Given that a photon excitation was emitted in state $|\lambda_1, \delta_1, x_1\rangle$, *reaching* the reflecting surface within a skin depth, and ending up in state $|\lambda_2, -\delta_1, x_2\rangle$, $\lambda_2 = \lambda_1$, the conditional amplitude density for the process is then given by $F(\mathbf{R}) = f(\mathbf{R})/\mathcal{A}(x_2, x_1)$, with $\int \mathrm{d}^2\mathbf{R} F(\mathbf{R}) = 1$, as a "summation" over impact centers whose nature will be now investigated.

Let $T_1 = x^0 - x_1^0$, $T_2 = x_2^0 - x^0$. The $\mathbf{r}-$, $z-$ integrals in (31) may be explicitly carried out yielding

$$\begin{aligned} F(\mathbf{R}) = \frac{1}{N} \sum_{\kappa_1,\kappa_2} \kappa_1\kappa_2 \int_0^\infty \mathrm{d}u_1 \int_0^\infty \mathrm{d}u_2 \frac{u_1 u_2}{\sigma\sqrt{1+\mathrm{i}(u_1+u_2)}} \\ \times \exp\left[-\frac{z_1^2}{\sigma^2}\frac{(u_1+\kappa_1\kappa_2 u_2 z_2/z_1)^2}{1+\mathrm{i}(u_1+u_2)}\right]\left[1+\mathrm{i}\frac{\sigma_o^2}{\sigma^2}(u_1+u_2)\right]^{-1} \\ \times \exp\left[-\frac{\sigma_o^2}{\sigma^4}\frac{[u_1(\mathbf{r}_1-\mathbf{R})+u_2(\mathbf{r}_2-\mathbf{R})]^2}{1+\mathrm{i}\sigma_o^2(u_1+u_2)/\sigma^2}\right]\mathrm{e}^{-\mathrm{i}G(u_1,u_2,\mathbf{R})/\sigma^2}, \end{aligned} \tag{32}$$

$$\begin{aligned} N = \sum_{\kappa_1,\kappa_2} \kappa_1\kappa_2 \int_0^\infty \mathrm{d}u_1 \int_0^\infty \mathrm{d}u_2 \frac{u_1 u_2}{\sigma\sqrt{1+\mathrm{i}(u_1+u_2)}} \\ \times \exp\left[-\frac{z_1^2}{\sigma^2}\frac{(u_1+\kappa_1\kappa_2 u_2 z_2/z_1)^2}{1+\mathrm{i}(u_1+u_2)}\right]\int \mathrm{d}^2\mathbf{r}\mathrm{e}^{-\mathrm{i}G(u_1,u_2,\mathbf{r})/\sigma^2} \end{aligned} \tag{33}$$

with

$$G(u_1, u_2, \mathbf{r}) = u_1[(\mathbf{r}_1-\mathbf{r})^2 + z_1^2 - T_1^2] + u_2[(\mathbf{r}_2-\mathbf{r})^2 + z_2^2 - T_2^2]. \tag{34}$$

For the practical case $\sigma \ll z_1$, with a given initially chosen macroscopic value z_1, i.e., for $z_1^2/\sigma^2 \gg 1$, we may use the distributional limit

$$\frac{z_1/\sigma}{\sqrt{1+\mathrm{i}(u_1+u_2)}} \exp - \frac{z_1^2}{\sigma^2}\frac{(u_1+\kappa_1\kappa_2 u_2 \frac{z_2}{z_1})^2}{1+\mathrm{i}(u_1+u_2)} \to \sqrt{\pi}\delta\left(u_1+\kappa_1\kappa_2 u_2\frac{z_2}{z_1}\right) \tag{35}$$

in (32) as obtained, for example, by Fourier transform techniques, to obtain $u_1 = -\kappa_1\kappa_2 u_2 z_2/z_1$, with the necessary restrictions $\kappa_1 = \pm 1$, $\kappa_2 = \mp 1$, giving for $\sigma \ll z_1$,

$$\begin{aligned} F(\mathbf{R}) = \frac{\mathrm{i}}{\pi\sigma_o^2 C} \int_0^\infty \frac{w^2 \mathrm{d}w}{\sigma_o + \mathrm{i}w} \exp\left[\frac{-\mathrm{i}wG(z_2, z_1, \mathbf{R}_o)}{\sigma_o^3(z_1+z_2)}\right] \\ \times \exp -\mathrm{i}\frac{w}{\sigma_o^2}\frac{(\mathbf{R}-\mathbf{R}_o)^2}{(\sigma_o+\mathrm{i}w)} \end{aligned} \tag{36}$$

where

$$C = \int_0^\infty w\mathrm{d}w \exp\left[\frac{-\mathrm{i}wG(z_2, z_1, \mathbf{R}_o)}{\sigma_o^3(z_1+z_2)}\right], \tag{37}$$

$$\mathbf{R}_o = (z_2\mathbf{r}_1 + z_1\mathbf{r}_2)/(z_1+z_2). \tag{38}$$

For an effective area of impact $\pi\sigma_o^2$ about a point $\mathbf{R}$ for $\sigma_o \to 0$, we obtain

$$\pi\sigma_o^2 F(\mathbf{R}) \to \exp -(\mathbf{R}-\mathbf{R}_o)^2/\sigma_o^2 \frac{\int_0^\infty w\mathrm{d}w \exp[\frac{-\mathrm{i}wG(z_2,z_1,\mathbf{R}_o)}{\sigma_o^3(z_1+z_2)}]}{\int_0^\infty w\mathrm{d}w \exp[\frac{-\mathrm{i}wG(z_2,z_1,\mathbf{R}_o)}{\sigma_o^3(z_1+z_2)}]} \tag{39}$$

with the second factor independent of σ_o, giving the remarkably simple expression

$$\pi\sigma_o^2 F(\mathbf{R}) \to \mathrm{e}^{-(\mathbf{R}-\mathbf{R}_o)^2/\sigma_o^2} \tag{40}$$

for $\sigma_o^2 \to 0$.

Accordingly, for an arbitrary small σ_o, giving a point-like area of impact, about the point $\mathbf{R}$, the partial amplitude $\pi\sigma_o^2 F(\mathbf{R})$ vanishes exponentially for $\mathbf{R} \neq \mathbf{R}_o$, i.e., for the non-classical point of impact. On the other hand for $\mathbf{R} = \mathbf{R}_o$, we have $\pi\sigma_o^2 F(\mathbf{R}) \to 1$ for $\sigma_o^2 \to 0$.

The condition $\mathbf{R} = \mathbf{R}_o$, translates from (38) to $(\mathbf{r}_1 - \mathbf{R}_o)/z_1 = -(\mathbf{r}_2 - \mathbf{R}_o)/z_2$ which is nothing but the law of reflection with $\mathbf{R}_o$ denoting the classical point of impact. For given $(\mathbf{r}_1, z_1)$, $(\mathbf{r}_2, z_2)$, leading from (38) to a fixed value of $\mathbf{R}_o$, we may choose the unit vectors $\mathbf{n}_+$, $\mathbf{n}_-$ in (11), to be directed along the vectors $(\mathbf{R}_o - \mathbf{r}_1, -z_1)$, $(\mathbf{r}_2 - \mathbf{R}_o, z_2) = z_2(\mathbf{R}_o - \mathbf{r}_1, z_1)/z_1$, respectively, corresponding to classical rays, with the vectors $(\mathbf{e}_1, \mathbf{e}_2)$, $(\boldsymbol{\epsilon}_1, \boldsymbol{\epsilon}_2)$ having the well know interpretations of polarization vectors perpendicular, respectively, to $\mathbf{n}_+, \mathbf{n}_-$, with transformations $\mathbf{e}_1 \leftrightarrow \boldsymbol{\epsilon}_1$, $\mathbf{e}_2 \leftrightarrow \boldsymbol{\epsilon}_2$ upon scattering.

Our formalism clearly opens the way for practical spacetime analyses of photon dynamics and *other* interacting field theories by using, in the process, functional differential techniques [10, 12] in different experimental situations. It shows, in particular, how amplitudes of propagation are determined from the knowledge of amplitudes of emissions and absorption of particle excitations by emitters and detectors, respectively, signaling the power of the present method of analysis. The time slicing procedure will also allow to derive path integrals for such amplitudes of propagation in spacetime. These further developments in general field theories emphasize the practicality and generality of the problem treated here not just being restricted to it. Such a program will be taken up in subsequent reports.

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